# Problem #9.10

**A popular children’s riddle is “Brothers and sisters I have none, but that man’s father is my father’s son.” Use the rules of family domain (Section 8.3.2 on page 301) to show who that man is. You may apply any of the inference methods described in this chapter. Why do you think that this riddle is difficult?**

This problem in prenex normal form is:

This is in conjunctive normal form.

The existential quantifier can be removed by making into a function of and . Hence:

Since there are only universal quantifiers remaining, these can be dropped resulting in:

The only value assignment to that will make this statement true is if is my son.

This riddle is not terribly difficult. However, it obfuscates by wrapping the object in what are compliementary operations since has no brothers.

# Problem #9.23

**From “Horses are animals,” it follows that “The head of a horse is the head of an animal.” Demonstrate that this inference is valid by carrying out the following steps:**

1. **Translate the premise and the conclusion into the language of first order logic. Use three predicates: (meaning “ is the head of ”), , and .**

The premise of this statement is “Horses are animals”. Rewritten in first-order logic with the defined predicates, this statement is:

The conclusion of this statement is:

1. **Negate the conclusion, and convert the premise and the negated conclusion into conjunctive normal form.**

By definition:

To perform refutation, negate the conclusion and show that:

The premise is already in prenex normal form so the quantifiers can be dropped resulting in:

This can be made into a single clause through implication elimination.

In the conclusion, the existential quantifier can be replaced by making a function of (i.e. ). Hence, the conclusion becomes:

Again, since all variables are bounded by a universal quantifier, the universal quantifier(s) can be dropped making the statement:

When implication elimination is applied to this equation, the result is:

To perform resolution refutation, the conclusion is negated. This results in:

The conjunction of the premise and the negation of the conclusion is taken. It results in:

This is in CNF format.